Social Welfare and Spatial Price Discrimination in Intermediate Good Markets

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Abstract

Holahan (1975, AER) sets up a linear market model for a final good in which consumers are uniformly distributed along a linear market and are price-discriminated against by a monopolist. It is found that social welfare is higher under discriminatory pricing than uniform mill pricing as the market area of the former is larger than the latter. This paper extends the Holahan framework to a vertically-related industry to examine the welfare effect of spatial price discrimination in an upstream market. It is found that the market area is larger, and output is higher, but social welfare is lower under discriminatory pricing than under uniform mill pricing. The upstream monopolist can serve a larger market area under discriminatory pricing, generating a positive welfare effect like that in Holohan (1975). However, this effect is less significant for price discrimination in an upstream market due to the existence of double marginalization, thereby making discriminatory pricing socially inferior to uniform mill pricing. Furthermore, this result is robust even if the production technology of the downstream firms exhibits non-constant returns to scale, as long as they are not returns to scale decreasing beyond a certain point.

JEL Classification: L12, L42
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1 Introduction

The effects on market outputs and social welfare of third-degree price discrimination have been discussed extensively in the literature. It is well known that the change in output resulting from third-degree price discrimination in final good markets is positively related to social welfare. The literature along this line of research can be dated back to Robinson (1933) who first proposed the criterion of "adjusted concavity" to examine the effect of third-degree price discrimination on output in a model with two separated markets. The linkage between output and social welfare is further explored by Schmalensee (1981) and Varian (1985). Compared to uniform pricing, an increase in output as a result of discriminatory pricing is proved to be a necessary condition for higher social welfare. This result is then challenged by Hwang and Mai (1990). Using a barbell model in which the location choice of a monopolist is endogenized, they show that price discrimination definitely lowers output, but may increase social welfare under some parameter combinations.

In recent years, the welfare analysis of third-degree price discrimination has been extended to intermediate good markets, see for example, Katz (1987), DeGraba (1990), Yoshida (2000), and Valletti, (2003), among others. They all show that price discrimination in intermediate good markets definitely lowers social welfare as the upstream monopolist, while engaging in price discrimination, tends to sell more output to less efficient downstream firms, causing efficiency loss in terms of social welfare. It is worth noting that Inderst and Valletti (2009) tackle the same issue, but from a different perspective, and derive an opposite result. They assume that the input monopolist faces a threat of demand-side substitution and find that the price discrimination the in input market can enhance social welfare under certain reasonable conditions.

On the other hand, the welfare comparison between uniform pricing and discriminatory pricing has also been analyzed in spatial frameworks. Greenhut and Ohta (1972) show that the total output is greater under spatially discriminatory pricing than under uniform mill pricing. More recently, Holahan (1975) has assumed that the market area of a spatial monopolist is endogenously determined and finds that the larger market area (and output) under spatial price discrimination leads to a higher level of social welfare. By contrast, Beckmann (1976) assumes the market area to be fixed and finds that spatially discriminatory pricing yields the same output, but lower social welfare, relative to uniform mill pricing.

To the best of our knowledge, the welfare effect of price discrimination for an intermediate good industry has not been investigated in a spatial framework *à la* Greenhut and Ohta (1972), Holahan (1975) and Beckmann (1976). The purpose of this paper is to fill this gap in the literature. We shall employ a linear market model, similar to Holahan (1975), to examine the welfare implications of spatially discriminatory pricing. In our model, there is one upstream monopolist which sells an intermediate good to the downstream firms that are distributed evenly along the linear market. Each downstream firm sells the final output to consumers in its respective final good market. The upstream monopolist can adopt either discriminatory pricing or uniform mill pricing when selling the intermediate good to the downstream firms.

It is found that the market area is larger, output is higher but social welfare is lower under discriminatory pricing than under uniform mill pricing. The upstream monopolist can serve a larger market area under discriminatory pricing, generating a positive welfare effect like that in Holohan (1975). However, this effect is less significant for price discrimination in an upstream market due to the existence of double marginalization, thereby making discriminatory pricing socially inferior to uniform mill pricing. This result is contrary to the finding in Holahan (1975), in which price discrimination in a final good market is definitely welfare-improving. This conclusion can be extended to the case with non-constant returns-to-scale technology. It will be shown that our result derived under constant returns-to-scale technology is still robust under variable returns to scale, as long as the production technology of the downstream firms does not exhibit sufficiently decreasing returns to scale.

The remainder of the paper is organized as follows. Section 2 introduces the basic model. The price decision and market area of an upstream monopolist adopting either spatially discriminatory or mill pricing are derived and compared in Section 3. In Section 4, we compare the level of social welfare under the two pricing regimes. Section 5 extends the basic model to the case in which the production function of each downstream firm exhibits decreasing or increasing returns to scale to examine whether our results derived previously hold true in a more general setting. Section 6 concludes the paper.

2 The basic model

Suppose that there is a linear spatial market where each point on the line is a sub-market with the following final good demand curve $P_x = a - bQ_x$, where $a > 0, b > 0$, the subscript x denotes the distance from the submarket to the location of an upstream monopolist, and P_x and Q_x are the corresponding market price and output, respectively. Each sub-market is served by only one downstream monopolist (i.e., there is no competition among the downstream producers). Assume further that the upstream monopolist is located at the center of the linear market, producing and selling an intermediate good to all the downstream firms.

In order to produce one unit of the final good, each downstream producer needs to acquire from the upstream monopolist one unit of the intermediate good.[1] Specifically, the downstream firm located in sub-market x pays an input price w_x and incurs a transport cost tx to obtain a unit of the intermediate good from the upstream monopolist, where t is the transport rate. In addition to the price of the intermediate good and transport cost, each downstream firm also has to incur a constant cost c to produce or market one unit of the final good. Suppose that the marginal production cost of the upstream monopolist is zero for simplicity without loss of generality. The upstream monopolist can adopt either uniform mill pricing or discriminatory pricing. Since the distribution of the downstream firms in the linear market is symmetrical to the location of the monopolist, we need to consider only one side of the market. In what follows, we shall assume that the upstream monopolist is located at point 0 of the linear market and consider its right-hand-side markets only. That is, $x \in [0, \infty]$.

The game consists of two stages. In the first stage, the upstream monopolist chooses either discriminatory pricing or mill pricing to determine the optimal prices of its intermediate good, in anticipation of the equilibrium in the second stage. In the second stage, each downstream producer, given the price of the intermediate good offered by the upstream monopolist, chooses its optimal output (or price) for its local market. Given the specifications of the model, the profit function of the downstream firm in market x is defined as follows:

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[1] It is assumed that the production technology is of a fixed-coefficient technology, which is a common assumption in the literature on input price discrimination. See, for example, Katz(1987), Degraba (1990) and Yoshida (2000).

$$\pi_x = (P_x - w_x - tx - c)Q_x, \quad (1)$$

where π_x is the profit of the downstream firm and w_x is the mill price of the intermediate good. By differentiating (1) with respect to Q_x, we can derive the first-order condition for profit maximization for the downstream monopolist as follows:

$$a - 2bQ_x = w_x + c + tx,$$

where the left-hand (right-hand) side of the above equation is the marginal revenue (the marginal cost). It is straightforward to derive the equilibrium quantity and price in sub-market x for the second stage of the game as follows:

$$Q_x(w_x) = \frac{a - (c + w_x + tx)}{2b}. \quad (2)$$

$$P_x(w_x) = \frac{a + c \ + w_x + tx}{2}. \quad (3)$$

Note that (2) is also the derived demand of the downstream firm located at x. We shall utilize this derived demand to solve for the optimal price of the intermediate good in the next section.

3 The optimal prices of the upstream monopolist

3.1 Equilibrium under uniform mill pricing

In this section, we first investigate the optimal price of the upstream monopolist if it engages in uniform mill pricing. Given the derived demand curve in (2), the upstream monopolist sets a uniform input price w^m for all the downstream firms to maximize its profit Ω^m which is specified as follows:

$$\Omega^m = w^m \int_0^{x^m} \frac{a - (c + w^m + tx)}{2b} dx, \quad (4)$$

where Ω^m denotes the profit of the upstream monopolist, the superscript "m"

indicates that the variables are associated with mill pricing, and x^m is the corresponding boundary of the market area served by the upstream monopolist, which can be derived by setting the output in (2) equal to zero:

$$x^m = \frac{a-c-w^m}{t}. \tag{5}$$

By substituting (5) into (4) and then differentiating it with respect to w^m, we can obtain the first-order condition for profit maximization for the upstream monopolist as follows:

$$\frac{\partial \Omega^m}{\partial w^m} = \frac{(a-c-w^m)(a-c-3w^m)}{4tb} = 0, \tag{6}$$

The corresponding second-order condition requires the second derivative of the profit function to be negative. That is:

$$\frac{\partial^2 \Omega^m}{\partial w^{m2}} = -\frac{2(a-c)-3w^m}{2tb} < 0. \tag{7}$$

By solving (6), we can derive the equilibrium input price w^m as follows:

$$w^m = \frac{a-c}{3}, \tag{8}$$

By substituting (8) into (2) and (3), we can derive the equilibrium output and price for the final good market at x and the market boundary as follows:

$$Q_x^m = \frac{2a-2c-3tx}{6b}, \forall x \in [0, x^m], \tag{9}$$

$$P_x^m = \frac{2a+c}{3} + \frac{tx}{2}, \forall x \in [0, x^m] \tag{10}$$

and

$$x^m = \frac{2(a-c)}{3t} \tag{11}$$

3.2 Equilibrium under discriminatory pricing

By proceeding as before, we can also derive the equilibrium under discriminatory pricing. If the monopolist engages in spatially discriminatory pricing, it charges the downstream firm in sub-market x an individual price w_x^d and has the following profit function:

$$\Omega^d = \int_0^{x^d} \frac{w_x^d (a - c - w_x^d - tx)}{2b} dx, \tag{12}$$

where variables with a superscript "d" indicate that the variables are associated with the discriminatory pricing regime, and x^d is the corresponding boundary of the market area served by the upstream monopolist.

Under the discriminatory pricing regime, the upstream monopolist can charge each sub-market an optimal price, so that the profit-maximization problem of the upstream monopolist can be solved by choosing w_x^d for each sub-market x. The profit from sub-market x is specified as follows:

$$\Omega_x^d = \frac{w_x^d (a - c - w_x^d - tx)}{2b}. \tag{13}$$

Differentiating (13) with respect to w_x^d, we can obtain the first-order condition for profit maximization as follows:

$$\frac{\partial\, \Omega_x^d}{\partial w_x^d} = -\frac{a - c - 2w_x^d - tx}{2b} = 0 \tag{14}$$

Note that the second-order condition is satisfied as the second derivative is necessarily negative:

$$\frac{\partial^2 \Omega_x^d}{\partial w_x^{d2}} = -\frac{1}{b} < 0,\ \forall x \in [0, x^d]. \tag{15}$$

Solving (14) yields the equilibrium input price, w_x^d, as follows:

$$w_x^d = \frac{a - c - tx}{2}, \forall x \in [0, x^d]. \tag{16}$$

From this equilibrium input price, we can then solve for the equilibrium output and price of the final good in sub-market x and the market boundary as follows:

$$Q_x^d = \frac{a-c-tx}{4b}, \forall x \in [0, x^d], \quad (17)$$

$$P_x^d = \frac{3a+c+tx}{4}, \forall x \in [0, x^d] \quad (18)$$

and

$$x^d = \frac{a-c}{t}. \quad (19)$$

By using the equilibrium outcomes in Section 3, we can compare the welfare between discriminatory pricing and mill pricing in the next section.

4 A welfare comparison between discriminatory and uniform mill pricing

Holahan (1975) considers price discrimination in a final good industry and shows that a monopolist can acquire a larger market area under discriminatory pricing than mill pricing, and because of this larger market effect, discriminatory pricing is superior to mill pricing in terms of social welfare. However, this outcome no longer holds for an intermediate good industry.

Before comparing the difference in welfare between the two pricing regimes, we first compare the market area and the changes in input price and market output in each sub-market between discriminatory pricing and mill pricing. First of all, from (11) and (19), it is straightforward to show that the market area is larger under discriminatory pricing than under mill pricing, that is:

$$x^m = \frac{2}{3}x^d < x^d. \quad (20)$$

This result is quite intuitive and similar to the finding in Holahan (1975). Under discriminatory pricing, a monopolist will charge each submarket an individual price and serve the markets up to the point at which its profit from an additional market becomes negative, which is of course higher than that under mill pricing.

We can now move to the comparison of the input prices. Let us denote Δw_x as the difference in input prices between discriminatory pricing and uniform mill pricing in sub-market x, which is derivable as follows:

$$\Delta w_x = w_x^d - w^m = \frac{(a-c)-3tx}{6} \gtreqless 0 \quad if \quad x \lesseqgtr \frac{a-c}{3t}, \forall x \in [0, \frac{2(a-c)}{3t}]. \tag{21}$$

As shown in (21), among the downstream firms which are served by the upstream monopolist under either pricing policy, the first half of the firms, which are more adjacent to the upstream monopolist, pay a higher price and the second half pay a lower price under discriminatory pricing than uniform mill pricing. In addition, the average input prices in this region are the same between the two pricing schemes which can be easily verified by integrating (21) from 0 to x^m.

We now turn to compare the market output between the two pricing policies. Using (9) and (17), we can draw Figure 1 as follows:

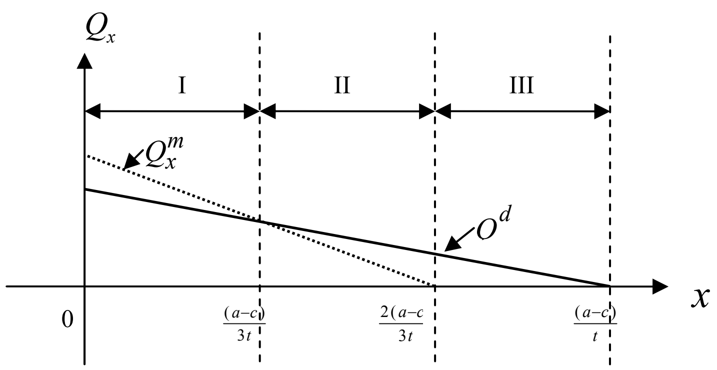

Figure 1: The output schedules of the two pricing policies

Figure 1 divides the market area into three regions − regions I, II and III. Note that both regions I and II are served by the upstream monopolist under either discriminatory or uniform mill pricing, while region III is served only under discriminatory pricing. As shown in the figure, the output is lower under discriminatory pricing than mill pricing in region I, as the upstream monopolist, in order to extract the maximum rent from its downstream firms, charges a higher input price, or supplies a smaller amount of the input, to its more adjacent downstream firms under discriminatory pricing as shown in (21). As shown in the figure, the output difference ($Q_x^d - Q_x^m$) increases with x due to a gradually lower input price under discriminatory pricing, turning from negative to positive in region II. Moreover, in region III, we observe a positive output difference (the output under uniform mill pricing is zero there), but it declines with distance due to the lower derived demand caused by a higher transport cost.

Let us now turn our attention to the comparison of the total outputs in regions I and II, which is the market area served by the upstream monopolist under both pricing policies. From (9) and (17), it is straightforward to show that discriminatory pricing yields the same total output, relative to uniform mill pricing. This result is similar to that concluded in Beckmann (1976), who employs a model similar to Holahan (1975), but assumes that the market area is fixed and the same under the two pricing policies. He shows that the total outputs under the two pricing policies are the same.

By summarizing these findings and taking into account the output in region III, we can establish the following proposition.

Proposition 1 *The market area and the total output of the final good are both larger under discriminatory pricing than uniform mill pricing.*

In order to compare the welfare levels of the two pricing policies, we first of all use the equilibria derived in the previous sections to calculate the consumer's surplus, profits of the downstream firm and the upstream monopolist, and then the social welfare in sub-market x under the two pricing policies. In the case of mill pricing, they can be derived as follows:

$$cs_x^m = \frac{(2a-2c-3tx)^2}{72b} \tag{22}$$

$$\pi_x^m = \frac{(2a-2c-3tx)^2}{36b} \tag{23}$$

$$\Omega_x^m = \frac{(a-c)(2a-2c-3tx)}{18b} \tag{24}$$

$$\begin{aligned} sw_x^m &= cs_x^m + \pi_x^m + \Omega_x^m \\ &= \frac{(10a-10c-9tx)(2a-2x-3tx)}{72b} \end{aligned} \tag{25}$$

The total social welfare under mill pricing can be derived from (25) as follows:

$$SW^m = \int_0^{x^m} sw_x^m \, dx = \frac{2(a-c)^3}{27tb}. \tag{26}$$

In the case of discriminatory pricing, we have the following:

$$cs_x^d = \frac{(a-c-tx)^2}{32b} \tag{27}$$

$$\pi_x^d = \frac{(a-c-tx)^2}{16b} \tag{28}$$

$$\Omega_x^d = \frac{(a-c-tx)^2}{8b} \tag{29}$$

$$\begin{aligned} sw_x^d &= cs_x^d + \pi_x^d + \Omega_x^d \\ &= \frac{7(a-c-tx)^2}{32b} \end{aligned} \tag{30}$$

and the total social welfare is:

$$SW^d = \int_0^{x^d} sw_x^d \, dx = \frac{7(a-c)^3}{96tb}. \tag{31}$$

By subtracting (26) from (31), we obtain the difference in social welfare between discriminatory pricing and uniform mill pricing as follows:

$$\Delta SW = SW^d - SW^m = \frac{-(a-c)^3}{864tb} < 0. \tag{32}$$

According to (32), the following proposition is established.

Proposition 2 *When the market area is endogenously determined, spatial price discrimination in an upstream market leads to lower social welfare, relative to uniform mill pricing, even though the market area of the former is larger.*

The result in the proposition can be well explained by the change in welfare in each sub-market. From (25) and (30) the difference in welfare in sub-market x is defined as follows:

$$\Delta sw_x = \begin{cases} sw_x^d - sw_x^m & if\ x \le x^m \\ sw_x^d & if\ x^m < x \le x^d \end{cases}$$

$$= \begin{cases} \dfrac{-(a-c-3tx)[17(a-c)-15tx]}{288b} & if\ 0 \le x \le x^m \\ \dfrac{7(a-c-tx)^2}{32b} & if\ x^m < x \le x^d \end{cases} \tag{33}$$

From (33), we can draw Figure 2 which shows the difference in welfare in each sub-market.

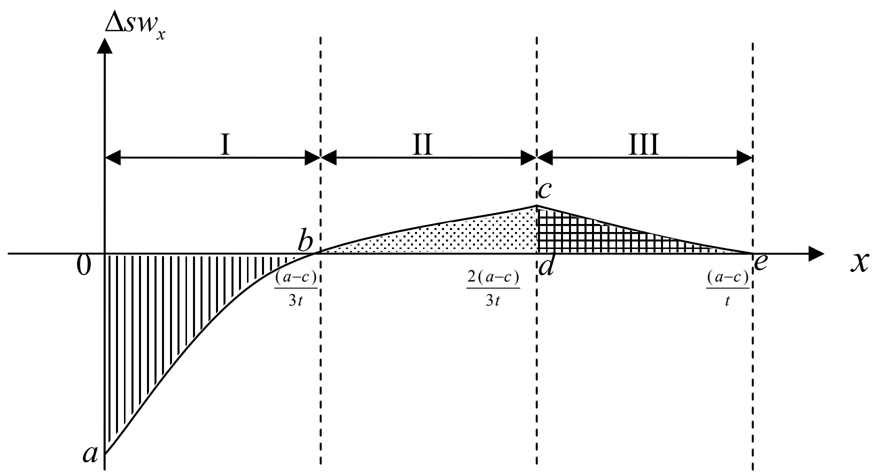

Figure 2: The change in welfare in each sub-market

As mentioned before, regions I and II comprise the market area served by the monopolist under either form of pricing. In region I, each downstream firm pays a higher input price resulting in lower market output and social welfare under discriminatory pricing than uniform mill pricing. The welfare loss from discriminatory pricing in region I is measured by the area $0ab$ in the Figure. In region II, the price of the input becomes lower and therefore the output and the welfare become higher under discriminatory pricing. The welfare gain from discriminatory pricing in this region is measured by the area bcd. Since the area $0ab$ is larger than the area bcd, there is a net welfare loss ($0ab - bcd$) in the market area from 0 to x^m (region I and region II together). This result is similar to that in Beckmann (1976) in which the market area for a final good monopolist is exogenously given and the output levels are the same while welfare is lower under discriminatory pricing than uniform mill pricing. Moreover, there is region III which is served under discriminatory pricing only, and the welfare gain from this region is represented by the area cde. In Holahan (1975), the welfare gain from the extra markets (region III) is more than enough to compensate for the welfare loss from regions I and II together, making discriminatory pricing a desirable pricing policy. However, this is not the case when price discrimination takes place in an input market. If the upstream monopolist adopts discriminatory pricing, the welfare gain arising from the extra market area (i.e., region III in our model) contracts, but the welfare loss from the commonly-served regions (i.e., regions I and II in our model) expands due to the existence of double marginalization, turning discriminatory pricing into a socially undesirable policy.

To signify the importance of double marginalization in the welfare calculation, let us purposely assume that each downstream firm has no market power in its local market and that the market price of the final good is equal to its marginal cost. As a result, there is no double marginalization. Under such circumstances, the output of the final good in each submarket is determined by setting price equal to marginal cost:

$$a - bQ_x = w_x + tx + c$$

Furthermore, the derived demand perceived by the upstream monopolist becomes:

$$\tilde{Q}_x = \frac{a - tx - c - w_x}{b}, \tag{34}$$

By proceeding as before, we can derive the social welfare in market x for uniform mill and discriminatory pricing, respectively, as follows:

$$\widetilde{sw}_x^m = \frac{(4a - 4c - 3tx)(2a - 2x - 3tx)}{18b}, \tag{35}$$

$$\widetilde{sw}_x^d = \frac{3(a - c - tx)^2}{8b}. \tag{36}$$

From (35) and (36), it is straightforward to derive the total social welfare under the two pricing policies and their difference is as follows:

$$\Delta \widetilde{SW} = \widetilde{SW}^d - \widetilde{SW}^m = \frac{(a-c)^3}{8tb} - \frac{10(a-c)^3}{81tb} = \frac{(a-c)^3}{648tb} > 0. \tag{37}$$

Hence, if each downstream firm has no monopolistic power in its own market, the welfare outcome becomes the same as that in Holahan (1975). That is, spatial price discrimination is welfare-improving.

5 Price discrimination and returns to scale

In this section, we shall retain the double marginalization assumption, but relax the assumption of constant production (marketing) cost by allowing returns to scale to vary. We shall prove that Proposition 2 is still robust as long as the technology of the downstream firms does not exhibit sufficiently decreasing returns to scale.

To introduce the property of returns to scale in our model, we assume that the marginal cost function of the downstream firms takes the following form:[2] $fQ + c$, where Q is the output of a downstream firm with $0 < c < a$. The production technology exhibits decreasing (constant, increasing) returns to scale if $f > (=, <)\, 0$.[3]

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[2] This marginal cost function is borrowed from San and Stamatopoulos (2009)

[3] In the model of San and Stamatopoulos (2009), it is assumed that the marginal cost function of the

Given this setting, the profit function of the downstream firm in sub-market x is redefined as:

$$\pi_x = (P_x - w_x - tx - c - fQ_x)Q_x$$

Then, the derived demand for the intermediate good in sub-market x is rewritten as follows:

$$Q_x(\omega_x) = \frac{a-(c+w_x+tx)}{2(b+f)}.$$

By proceeding as in Section 3, we can solve for input prices and market areas under the two pricing schemes. It is found that the input price and market area are identical to those obtained in the case with constant returns-to-scale technology. Therefore, by using (8), (11), (16) and (19) and the above derived demand, we can then recalculate the market output, consumer's surplus, profits earned by the downstream firm and the upstream monopolist and the social welfare in sub-market x under discriminatory or mill pricing. For the case of mill pricing, these items may be derived as follows:

$$Q_x^m(f) = \frac{2a-2c-3tx}{6(b+f)}, \forall x \in [0, x^m],$$

$$cs_x^m(f) = \frac{b(2a-2c-3tx)^2}{72(b+f)^2},$$

$$\pi_x^m(f) = \frac{(2a-2c-3tx)^2}{36(b+f)},$$

$$\Omega_x^m(f) = \frac{(a-c)(2a-2c-3tx)}{18(b+f)}$$

and

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firm is as follows: $\max\{fQ+c, 0\}$. When $f<0$, the marginal cost function is given as:

$$mc(Q) = \begin{cases} fQ+c & \text{if } Q \le -c/f \\ 0 & \text{if } Q > -c/f \end{cases}$$

The production technology exhibits increasing returns to scale only if the following condition is satisfied: $Q \le -c/f$. We shall assume this is the case throughout the paper.

$$sw_x^m(f) = \frac{[(10b+8f)(a-c)-(9b+6f)tx](2a-2x-3tx)}{72(b+f)^2}. \quad (38)$$

For the case of discriminatory pricing, we have:

$$Q_x^d = \frac{a-c-tx}{4(b+f)}, \forall x \in [0, x^d],$$

$$cs_x^d(f) = \frac{b(a-c-tx)^2}{32(b+f)^2},$$

$$\pi_x^d(f) = \frac{(a-c-tx)^2}{16(b+f)},$$

$$\Omega_x^d(f) = \frac{(a-c-tx)^2}{8(b+f)},$$

and

$$sw_x^d(f) = \frac{(7b+6f)(a-c-tx)^2}{32(b+f)^2}. \quad (39)$$

The difference in social welfare between the two pricing schemes can be derived from (38) and (39) as follows:[4]

$$\Delta SW(f) = \int_0^{x^d} sw_x^d(f)dx - \int_o^{x^m} sw_x^m(f)dx = \frac{(a-c)^3(2f-3b)}{2592tb}, \quad (40)$$

where x^m and x^d are defined as in (11) and (19), respectively.

From (40), it is clear that if $f \le 0$, where the technology exhibits non-decreasing returns to scale, the social welfare at x is definitely lower under discriminatory pricing than mill pricing. By contrast, for $f > 0$, i.e., when the production technology of each downstream firm exhibits decreasing returns to scale, the sign of $\Delta SW(f)$ is ambiguous as it depends on the degree of the decreasing returns to scale. It is positive only if f is larger than $3b/2$, i.e., the technology must exhibit sufficiently decreasing returns to scale. From the above discussion, we can establish the following proposition:

[4] We shall assume, throughout the rest of the analysis, $b+f>0$ to ensure that the total market output is positive.

Proposition 3 *With an endogenously determined market area, discriminatory pricing by an upstream monopolist is more socially desirable than uniform mill pricing only if the production technology of the downstream firms exhibits sufficiently decreasing returns to scale.*

To better explain the intuition behind the above proposition, we rewrite the welfare difference in market x as follows:

$$\Delta sw_x(f) = \begin{cases} sw_x^d(f) - sw_x^m(f) & if\ 0 \le x \le x^m \\ sw_x^d(f) & if\ x^m < x \le x^d \end{cases}$$

$$= \begin{cases} \dfrac{-(a-c-3tx)[17b(a-c)+10f(a-c)-(15b+6f)tx]}{288(b+f)^2} & if\ 0 \le x \le x^m \\ \dfrac{(a-c-tx)^2(7b+6f)}{32(b+f)^2} & if\ x^m < x \le x^d \end{cases} \tag{41}$$

Differentiating (41) with respect to f, we have:

$$\frac{\partial \Delta sw_x(f)}{\partial f} \begin{cases} > 0 & if\ x < \dfrac{a-c}{3t} \\ = 0 & if\ x = \dfrac{a-c}{3t} \\ < 0 & if\ \dfrac{a-c}{3t} < x < \dfrac{a-c}{t} \end{cases}. \tag{42}$$

By applying this result to Figure 2 which represents the welfare difference function with $f = 0$, we can draw a new welfare difference function $a'bc'e$ for an increase in f from zero as shown in Figure 3.

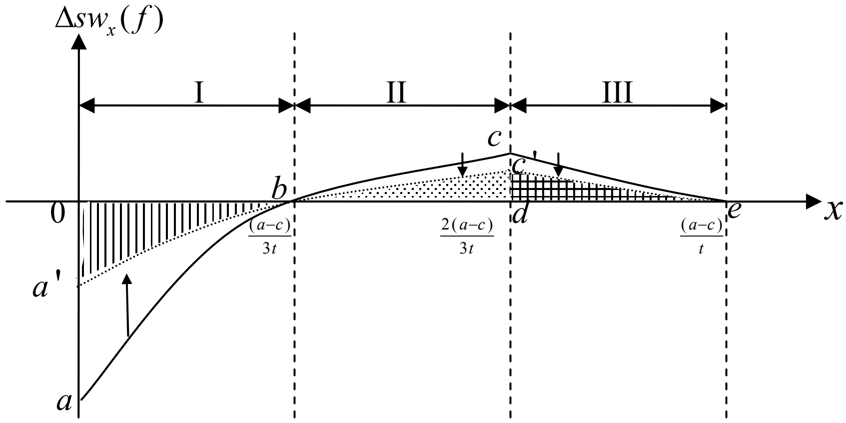

Figure 3: Returns to scale and change in welfare

We have shown that discriminatory pricing always lowers social welfare if the production technology exhibits non-decreasing returns to scale (i.e., $f \leq 0$), but it becomes ambiguous if it exhibits decreasing returns to scale (i.e., $f > 0$). In what follows, we shall use Figure 3 to illustrate how a marginal increase in f affects the welfare in each sub-market. The new welfare difference function $a'bc'e$ is for a positive f while the old welfare difference curve $abce$ is for $f = 0$. As shown in the figure, for a positive f, the total welfare difference for regions I and II is still negative. However, due to the decreasing returns-to-scale technology, the net welfare loss in the two regions as a whole decreases because of the decrease in the total production costs of downstream firms located in the regions. In addition, the welfare difference in region III is still positive but decreases with f (from cde to $c'de$) due to a higher production cost resulting from a higher f. However, the increase in production cost caused by a higher f is relatively low in region III as the output of the downstream firm in that region is relatively small. Therefore, if the upstream monopolist adopts discriminatory pricing and the extent of the decreasing returns to scale is sufficiently high, the welfare gains from region III can thus compensate for the welfare loss in regions I and II, making discriminatory pricing more desirable than uniform mill pricing.

6 Concluding Remarks

This paper sets up a spatial market model similar to Holahan (1975) to examine the welfare implications of discriminatory pricing by an upstream monopolist. It is found that discriminatory pricing is definitely inferior to uniform mill pricing, even though the former can serve a larger market area and more consumers. The result implies that a larger market area (or greater output) is not a sufficient condition for spatially discriminatory pricing to be welfare-improving. These results are contrary to the findings of Holahan (1975). Moreover, if we allow the production technology of the downstream firms to exhibit variable returns to scale, the above-mentioned welfare ranking is still robust as long as the technology does not exhibit returns to scale that decrease beyond a certain level.

In a non-spatial framework, Schmalensee (1981) demonstrates that an increase in output is a necessary condition for third-degree price discrimination in a final good market to be welfare-improving. Our findings suggest that his criterion is robust even if space is taken into consideration or the discrimination takes place in an intermediate good market.

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